

BER of Alamouti Coded System:

$$\text{SNR} = \frac{\|h\|^2 P}{2\sigma^2}$$

$$= \|h\|^2 \cdot \frac{1}{2} \cdot \frac{P}{\sigma^2}$$

$$= \|h\|^2 \cdot \frac{1}{2} \cdot \text{SNR}$$

BER at high SNR

$$= 2^{L-1} C_L \cdot \left(\frac{1}{2 \cdot \frac{1}{2} \text{SNR}} \right)^L$$

$$L = 2$$

Average

$$= 3 C_2 \cdot \left(\frac{1}{\text{SNR}} \right)^2$$

BER of Alamouti Coded Wireless System

$$\text{BER} = \frac{3}{\text{SNR}^2}$$

h_1, h_2 are i.i.d Rayleigh
average power = 1.

Example of Alamouti Processing:

$$y = \underbrace{[2+j]}_{h_1} \underbrace{[1-2j]}_{h_2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

$$h_1 = 2+j$$

$$h_2 = 1-2j$$

1x2
MISO system

First Transmit Period.

$$y_1 = \begin{bmatrix} 2+j & 1-2j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

During Second Transmit Instant

$$y_2 = \begin{bmatrix} 2+j & 1-2j \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + w_2$$

$$= -(2+j)x_2^* + (1-2j)x_1^* + w_2$$

$$y_2 = -(2+j)x_2^* + (1-2j)x_1^* + w_2$$

$$y_2^* = -(2-j)x_2 + (1+2j)x_1 + w_2^*$$

$$= \begin{bmatrix} 1+2j & -2+j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_2^*$$

$$y_1 = \begin{bmatrix} 2+j & 1-2j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}}_{\tilde{y}} = \begin{bmatrix} \tilde{a}_1 & \tilde{a}_2 \\ 1+2j & -2+j \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix} w_1 \\ w_2^* \end{bmatrix}}_{\tilde{w}}$$

$$\bar{C}_1 = \begin{bmatrix} 2+j \\ 1+2j \end{bmatrix} \quad \bar{C}_2 = \begin{bmatrix} 1-2j \\ -2+j \end{bmatrix}$$

$$\bar{C}_1^H \bar{C}_2 = \begin{bmatrix} 2-j & 1-2j \end{bmatrix} \begin{bmatrix} 1-2j \\ -2+j \end{bmatrix}$$

$$= (2-j)(1-2j) + (1-2j)(-(2-j))$$

$$= (2-j)(1-2j) - (1-2j)(2-j)$$

$$= 0$$

To decode x_1 , perform

$$\frac{\bar{c}_1^H}{\|\bar{c}_1\|} \cdot \bar{y}$$

$$\bar{c}_1 = \begin{bmatrix} 2+j \\ 1+2j \end{bmatrix}$$

$$\|\bar{c}_1\| = \sqrt{|2+j|^2 + |1+2j|^2}$$

$$= \sqrt{4+1+1+4}$$

$$= \sqrt{10}$$

$$\frac{\bar{c}^H}{\|\bar{c}\|} \cdot \bar{y}$$

$$= \frac{1}{\sqrt{10}} \begin{bmatrix} 2-j & 1-2j \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}$$

$$= \frac{1}{\sqrt{10}} (2-j)y_1 + \frac{1}{\sqrt{10}} (1-2j)y_2^*$$

Extract x_1

Similarly to extract x_2 , Perform

$$\frac{\bar{c}_2^H}{\|c_2\|} \cdot \bar{y} \quad \|c_2\| = \sqrt{10}$$

$$\begin{aligned} &\rightarrow \frac{1}{\sqrt{10}} \begin{bmatrix} 1+2j & -2-j \end{bmatrix} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} \\ &= \frac{1}{\sqrt{10}} (1+2j)y_1 - \frac{(2+j)}{\sqrt{10}} y_2^* \end{aligned}$$

Extract x_2 .

Example for BER of Alamouti System:

Example: 1×2 MISO system
with Alamouti Coded Transmission
 $SNR = 25 \text{ dB}$
 $BER = ?$

$$\text{Avg BER} = \frac{3}{\text{SNR}^2}$$

$$10 \log_{10} \text{SNR} = 25$$

$$\text{SNR} = 10^{2.5}$$

$$\begin{aligned} \text{BER} &= \frac{3}{\text{SNR}^2} = \frac{3}{(10^{2.5})^2} \\ &= \frac{3}{10^5} = 3 \times 10^{-5} \end{aligned}$$

Avg BER
of Alamouti
at SNR = 25 dB.